References


**PLANNING AND CONDUCTING INQUIRY BASED MATHEMATICS COURSE FOR FUTURE PRIMARY SCHOOL TEACHERS**

*Libuše Samková*

**Abstract**

In this contribution I would like to present some methodological issues related to a teaching experiment conducted under an educational research aiming at implementation of inquiry based mathematics education into university courses for future primary school teachers. The text introduces rudiments of inquiry based mathematics education, describes a one-year inquiry based teaching experiment which I prepared and conducted within a compulsory university course on arithmetic, and focuses on questions related to planning and performing such a course. The final part of the paper briefly mentions results of the education research associated with the experiment.

University of South Bohemia, Czech Republic; e-mail: lsamkova@pf.jcu.cz
Keywords: Future primary school teachers, future teacher education, inquiry based mathematics education, teaching experiment

Introduction

This paper is devoted to relation between inquiry based mathematics education (IBME) and university preparation of primary school teachers, a topic that was already reported at previous SEMT conference where we discussed it from the perspective of pre-service and in-service primary school teachers who had attended standard university courses on mathematics accompanied by a short instruction and self-study on IBME (Hošpesová et al., 2015). This time I will approach the topic from a different perspective that includes future primary school teachers attending a one-year experimental course on arithmetic that was entirely conducted in inquiry based manner, and the main focus of this paper will deal with questions related to planning and performing the course.

Issues reported here are a part of a larger educational research project supported by Czech Science Foundation named Enhancing mathematics content knowledge of future primary teachers via inquiry based education. The goal of the project is to implement inquiry based education into university courses on mathematics and didactics of mathematics for future primary school teachers, and observe how active participation in the courses can influence professional competences of project participants.

The impact that IBME might have on students was investigated by many educational researchers up to now. An extensive overview of the research was provided e.g. by Bruder and Prescott (2013) but none of the studies mentioned there focused on the impact that IBME might have on future teachers when implemented systematically during their university preparation.

Inquiry based (mathematics) education

Recently a lot has been said and written about inquiry based education as one of the important means of gaining new knowledge (Bruder and Prescott, 2013; Jiang and McComas, 2015; Savelsbergh et al., 2016). But the concept of inquiry is not recent in pedagogy: the term can be traced long way back to the work of Dewey (1938) who characterized it by means of transformations of indeterminate situations. Nowadays, inquiry based pedagogy is usually characterized as a way of teaching in which students are invited to work in ways similar to how scientists work (Artigue and Blomhøj, 2013) and to use procedures known from scientific inquiries: to observe, pose questions, reason, think, search for relevant information, collaborate, collect data and interpret them, solve and discuss problems that come out from real life or can be applied in everyday life contexts (Dorrier and Maaß, 2014). These procedures are naturally adapted to school context, so that during inquiry based mathematics lessons students do not discover new scientific issues but rediscover issues from school mathematics or solve
simple problems of everyday application character. The role of the teacher in the inquiry based mathematics lesson consists mainly in creating a suitable environment, building upon students' reasoning, giving students support, and in connecting to students' experience (Dorrier and Maaß, 2014).

In mathematics education we can find various frameworks that employ approaches similar to IBME, many of them did it long before the term IBME has appeared: problem solving (Pólya, 1945), theory of didactical situations (Brousseau, 1997), realistic mathematics education and guided rediscovery (Freudenthal, 1973), mathematical modelling (Kaiser-Meßmer, 1986), substantial learning environments (Wittmann, 2001), etc. Also in the Czech educational past we can find issues related to the concept of inquiry, e.g. the concept of strengthening contacts of mathematics education with everyday reality and with other school subjects, and the concept of the process of grasping situations that were discussed by Koman and Tichá (1998), or the concept of experiments in the mathematical classroom (Hruša and Vyšín, 1964).

**IBME in the classroom**

In the mathematics classroom, the starting point for inquiry activities of students consists in creating an appropriate learning environment, usually in the form of a task or a problem that the students have to solve. To stimulate inquiry activities of students, the task should contain something unknown for the solver what is perceived by the solver as thought-provoking or interesting. But this unknown part of the task should not be too distant from student's actual knowledge, because inquiry is possible only when the unknown part can be approached through something known – only known facts and their relations might lead to conjectures and judgements that allow the solver to seek the solution. Tasks that may lead to inquiry activities of students are called *inquiry tasks*. The employment of an inquiry task within the lesson does not automatically guarantee that inquiry activities of students really occur in the classroom – this can be achieved only when the difficulty of the task is suitably chosen with respect to the actual students' knowledge, and when the characteristics of IBME are met by the teacher as well as by the students.

From the perspective of the task design, inquiry tasks are usually *open* in the sense of open-approach to mathematics, i.e. at least one of the following parameters of the task is not exactly given: starting situation, process, end products, ways to develop (Nohda, 1995, 2000). So that, solving an open task may consist of various ways of formulating the task mathematically, of investigating various approaches to the formulated task, of various interpretations of the found results, and/or of posing various advanced tasks.

Nohda (2000) offers four dimensions that can be used to evaluate students' responses to open tasks:
Inquiry in the classroom may be classified in various manners; one of the most common is the classification according to the kind of information that the teacher provides the students (Banchi and Bell, 2012; Bruder and Prescott, 2013):

- confirmation inquiry – the teacher assigns the students a question, and an appropriate method, results are known, the students are asked to confirm the results and explain them to others;

- structured inquiry – the teacher assigns the students a question and an appropriate method, the students are asked to find results and explain them to others;

- guided inquiry – the teacher assigns the students a question, the students are asked to find an appropriate method and results, and explain them to others;

- open inquiry – students themselves pose questions, search for an appropriate method and results, and explain them to others.

As Bruder and Prescott (2013) present in their survey study, the greatest positive knowledge gains for the students in both content and process are documented in studies that monitor effects of guided inquiry. This is the reason why I choose the guided inquiry level for the experiment.

The experiment

The referred experimental course was intended for 33 future primary school teachers, university students of the second year of the five-year master degree study at the Faculty of Education.

The teaching experiment was conducted within a compulsory course on arithmetic which acquaints the students with introduction to logic, set theory, and number systems. The course lasts 28 weeks; each week of schooling consists of a one-hour lecture and a two-hour seminar. In the referred year the students were divided into two groups for the seminars, so that the two-hour seminar on arithmetic was conducted twice a week, once for each group.

For the purpose of the experiment, the course on arithmetic was entirely rearranged into inquiry-based manner on guided inquiry level. Being the course compulsory, the overall mathematical content of the course had to be preserved, and standard ways of assessment had to be used during the course (four standard
written tests, and two oral examinations). These requirements complicated the rearrangement, and made it really challenging.

The following text will address some of the aspects related to the rearrangement.

**The known and the unknown**

The first step in the process of planning an inquiry based mathematics course of a long-term character consisted in overviewing the syllabus of the course, in determining which subject matter would have to play the role of the known part, and which subject matter would become the object of inquiry activities.

The known part mainly included necessary definitions, axioms, etc. For instance, for the topic "introduction to set theory" I had to establish the meaning of the following words and the convention for their notation: "set", "element", "subset", "empty set", "union", "intersection", "complement", "difference", "Cartesian product", "Venn diagram", "cardinality". Such matter was presented at lectures.

The inquiry activities based on the known part then took place at seminars. For instance, at one of the seminars to the topic "introduction to set theory" the students themselves undertook inquiries on whether and how it was possible to utilize Venn diagrams during solving various types of word problems.

**Time constraints**

Along with the process of determining what will play the role of the known, I also had to pay attention to time constraints. I had to check whether all the inquiry activities planned for the course would get enough time to enable inquiry of decent quality.

Some types of the inquiry activities seem to be too time-consuming to be successfully implemented into the course but when we suitably choose the object of such an inquiry, the time devoted to the inquiry activities may be compensated by saving the time elsewhere. For example, when the inquiry results in discovering new subject matter, then no instruction on this subject matter may be needed any more.

As an illustration I will describe a part of one of the seminars devoted to the topic "introduction to set theory", namely to Venn diagrams. In the first part of the seminar the students were solving many tasks focusing on how to draw and read Venn diagrams to various sets and subsets and to various set operations. In the second part of the seminar, without any previous instruction on using Venn diagrams in solving word problems, the following word problem was assigned to the students as individual work:

Some children from our class went on holiday trip and visited Praha, Brno or Olomouc. Three boys went on the trip, one to each town. Jitka travelled to Brno

The students worked on the task quite a long time, more than 20 minutes, but this activity successfully replaced the original "non-inquiry" part of the seminar that had lasted 45 minutes the year before. The last-year "non-inquiry" activities had consisted in an instruction on using Venn diagrams in solving word problems, and in a graded series of word problems solved with help of Venn diagrams on the blackboard; the task about the children on holiday trip being the last in the order. Comparing the non-inquiry and inquiry approaches, we can see that the inquiry approach saved us 25 minutes. This saved time was devoted to whole-class discussion on the holiday task and on methods of solving the students had used, as well as to individual consultations with weaker students.

Some of the inquiry tasks do not allow compensating the time similarly as in the previous case, so that I had to find the needed time otherwise. I removed from seminars several passages devoted to training of calculations (e.g. calculation tasks on addition and subtraction of decimal numbers), and assigned them to students as homework.

As the standard assessment written tests that are compulsory for the course comprise all the related subject matter (including word problems solved with Venn diagrams, and calculation tasks), the tests gave me the feedback on how the students mastered the subject matter mediated through inquiry activities as well as through homework. There were no significant differences between the test results from years when the inquiry was not implemented and the test results from the year when the inquiry was implemented.

Types of inquiry tasks

At seminars I assigned various types of inquiry tasks to students. From the perspective of general content targets the tasks fell into four groups:

1) Tasks that employed the most recently acquired knowledge, and applied it in new contexts, e.g. when discovering new solution methods. The above mentioned holiday task belongs here.

2) Tasks that employed knowledge that had been acquired by students some time before, and offered a new view on it: by linking topics with their practical applications, by combining various topics in one multiple-topic task, etc. For instance, to gain a new perspective on divisibility, on properties of natural numbers operations, on the concept of decimal numeral system, and on notation in non-decimal numeral systems, I invited the students to work on a multiple-topic task consisting in discovering a

---

1 Jitka, Vlasta, Eva, Sylva, Dana and Alena are girly names.
rule specifying how to recognize even numbers in non-decimal numeral systems (for more details see Samková and Tichá, 2016a).

3) Tasks that prepared the students for gaining knowledge on a quite new topic (in that case, the seminar with the inquiry task preceded the lecture assigned to the topic). For instance, the lecture on the topic "equivalence of sets" was preceded by a seminar with a series of inquiry tasks that employed manipulative activities focusing on comparisons by matching, and on rudiments of combinatorics (for one of the tasks see Fig. 1).

![Figure 1: Take 3 cubes and 3 hearts of different colours, and use all of them to make pairs consisting of one cube and one heart. How many pairs do you get? How many different ways of pairing exist? Show all of them! Repeat with more cubes and hearts, and generalize.](image)

4) Tasks that actually responded to a difficulty with which the students were not able to deal. For instance, as a reaction on difficulties that the students faced on one of the seminars while solving tasks on properties of canonical decompositions of the second and the third powers of natural numbers, I added into the following seminar an inquiry task that first invited the students to search for all the possible decompositions of various given natural numbers into a product of two natural numbers, and to search for all divisors of the given numbers. Then the task invited the students to investigate relations between the decompositions and the lists of divisors for various given numbers provided the numbers are (or are not) powers. Such a task successfully helped to overcome the initial difficulties.

From the perspective of the amount of information given in the assignment, the two extremal cases were of special interest during inquiry based lessons:

1) Tasks with sparse starting information. These tasks are much indeterminate in their assignment, thus offer a lot of space for inquiry, a lot of possibilities that can be taken into account. For example: The sum of two unknown numbers is 10. How can these numbers look like? (Note that no particular numeral system is assigned to the addends.)

2) Tasks with dense starting information. These tasks contain a lot of facts or a lot of terminological terms in their assignment, and the solver must orientate in them. Such tasks prepare students for handling amounts of data.
and for reading dense (scientific) texts. For example: *What is the greatest product that can be obtained by decomposing number 10 into a sum of natural numbers and multiplying the addends of the sum?*

Particular inquiry tasks could be also put together to form a composite task. From the perspective of the inner structure, the composite tasks may be divided into three groups (for illustrative schemes see Fig. 2):

1) Tasks of hierarchical structure. These tasks are composites of two or more sub-tasks, where the end products of some of the sub-tasks can be utilized as a part of a starting situation of the other sub-tasks. For example:

   a) *The sum of two unknown natural numbers is 10. How can these numbers look like?*

   b) *Decompose number 10 into a sum of two natural numbers. What is the greatest product that you can obtain by multiplying these numbers?*

2) Tasks with dynamic starting. These tasks are composites of two or more sub-tasks with the same question, where every sub-task adds new starting information or new conditions to the task. For example:

   a) *Decompose number 10 into a sum of two natural numbers. What is the greatest product that you can obtain by multiplying these numbers?*

   b) *Decompose number 10 into a sum of three natural numbers. What is the greatest product that you can obtain by multiplying these numbers?*

   c) *Decompose number 10 into a sum of natural numbers. What is the greatest product that you can obtain by multiplying these numbers?*

   d) *How does the number of addends influence the greatest product?*

   e) *What changes if decomposing numbers 7, 8, 9, or 11?*

   f) *What changes if decomposing into a sum of rational or real numbers?*

   (The last sub-task is here just for illustration, since being too difficult for future primary school teachers.)

3) Tasks with dynamic ending. These tasks are composites of two or more sub-tasks with the same starting situation, where every sub-task adds a new question to the task. For example:

   a) *Decompose number 10 into a sum of two natural numbers. What is the greatest product that you can obtain by multiplying these numbers?*

   b) *Decompose number 10 into a sum of two natural numbers. What is the smallest product that you can obtain by multiplying these numbers?*

The decomposition tasks were inspired by Artigue and Baptist (2012, p. 7).
The above mentioned groups of tasks are not disjunctive; one inquiry task may belong to more of them. More about suitable inquiry tasks and their typology can be found in (Samková et al., 2015).

Instead of conclusion ... some partial results

As mentioned in the Introduction, the here referred course was realized as a part of a larger educational research project. During the whole course we continuously collected various data from course students: completed worksheets, standard written tests, written solutions to problems or tasks, reflections, etc. In our contributions related to the project we already discussed some aspects that we revealed in collected data. We presented:

- positive changes in students' approach to argumentation (a shift towards more efficient use of counter-examples, and a shift from using empirical arguments to attempts of using deductive arguments), for details see (Samková and Tichá, 2016a);

- positive changes in students’ open approach to mathematics (a shift towards seeking more or all solutions, and a shift towards accepting more forms of notations), for details see (Samková and Tichá, 2016b);

- changes in students' beliefs about mathematics and mathematics education (e.g. the newly emerged belief that discovering a thing by oneself helps remembering the thing, and also an ongoing change of the belief that...

Figure 2: Schemes of composite tasks – hierarchical task (left), task with dynamic starting (right top), task with dynamic ending (right bottom); schemes taken from (Samková et al., 2015), and translated
mathematics is about memorizing formulas and procedures), for details see (Samková and Tichá, 2016c).

These results indicate that inquiry based mathematics education might play an important role in university preparation of future primary school teachers. From my personal perspective, I would recommend this approach to other educators of future primary school teachers.

Acknowledgement:
This research was supported by the Czech Science Foundation, project No. 14-01417S.

References


PREPARING PRE-SERVICE TEACHERS FOR INCLUSIVE MATHEMATICS CLASSROOMS – CONCEPTS FOR PRIMARY EDUCATION

Petra Scherer

Abstract

In Germany inclusive education becomes more and more common for students with special needs. Teacher education programs preparing teachers for an inclusive school system are in the state of development at the moment. The project ProViel ("Professionalisation for Diversity") at the University of Duisburg-Essen aims at developing concepts and modules for teacher education for mathematics as well as for other subjects. The paper presents the project’s aims and objectives, followed by data concerning primary teacher students’ existing experiences and beliefs with respect to inclusive mathematics. Moreover, the design and try-out of a first teacher education course will be shown.

Keywords: Inclusive education, special education, teacher education programs, clinical interviews, substantial learning environments

Introduction

In Germany students with special needs either visit special schools for handicapped children or visit regular schools in inclusive settings. Both settings show extremely heterogeneous groups in classrooms and the teacher is confronted with individual handicaps, for example deficits in language or visual perception,

University of Duisburg-Essen, Germany; e-mail: petra.scherer@uni-due.de