Knowing how-to-act-in-the-moment in the classroom requires time for conceptual reflection about the merits of tasks and possible modifications to guide the mathematical experience of learners (Mason, 2002). The more relations the teacher constructs, the greater the potential to help students build up their mathematical experience and mathematical connections.

The teacher noticed that the format of the task gave no opportunity for inductive thinking, reason for which she developed sequences of diagrams, so that students could make the transition from particular cases to general cases. The teacher’s transformation of the task emerged from her own *interpretants*, mediated by visualization and diagrammatic reasoning, which in turn, allowed her to deepen her mathematical experience and to increase the potential for a richer mathematical experience for her students.

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**References**


**PREPARING FUTURE TEACHERS FOR FORMATIVE ASSESSMENT: THE CASE OF CONCEPT CARTOONS**

Libuše Samková

**Abstract**

In this contribution I will address the issues related to the use of formative assessment in inquiry based mathematics education. I will show how an educational tool called
Concept Cartoons may be used in university training of future teachers to prepare them for future implementation of formative assessment into their school practice. The text introduces the background of inquiry based mathematics education and Concept Cartoons that relates to formative assessment, and describes a small qualitative empirical study conducted with future primary school teachers. The study focuses on aspects related to on-the-fly formative assessment that appeared in future teachers’ written responses to Concept Cartoons and in a subsequent discussion.

**Keywords:** Concept Cartoons, formative assessment, future teacher education, inquiry based mathematics education

**Introduction**

This paper is devoted to relation among formative assessment, inquiry based mathematics education, university training of future teachers and Concept Cartoons. Partially, these topics have been repeatedly discussed at SEMT conferences: formative assessment from the perspective of students was explored by Martin, Wang, Lambert, Polly and Pugalee (2015), with relation to inquiry based mathematics education by Hošpesová, Stuchlíková and Žlábková (2017), questions related to inquiry based education from the perspective of future teachers were reported by Hošpesová, Samková, Tichá and Roubíček (2015), Samková (2017), Concept Cartoons were introduced by Samková and Naylor (2015), Samková, Tichá and Hošpesová (2015).

Issues reported here are a part of a larger educational research project supported by the Technology Agency of the Czech Republic named *Learning Hyperspace for Formative Assessment and Inquiry Based Science Teaching*. The goal of the project is to create a learning hyperspace for teachers where they could learn how to implement formative assessment into their inquiry based teaching. This particular paper belongs to the preparatory stage of the project in which we map the properties of the connection between inquiry based education and formative assessment and their possible impact on the form and content of the hyperspace, and also look for appropriate tools that could be used as a part of the hyperspace.

The reported empirical study aims to address the following research question: “Which aspects related to on-the-fly formative assessment appear in future teachers' written responses to Concept Cartoons and in a subsequent discussion?”

**Inquiry based (mathematics) education**

Inquiry based pedagogy is usually characterized as a way of teaching in which students are invited to work in ways similar to how scientists work (Artigue and Blomhøj, 2013). These procedures are naturally adapted to school context, so that students do not discover new scientific issues but rediscover school mathematics or solve simple problems of everyday application character. The role of the teacher in such lessons consists mainly in creating a suitable learning environment, building upon students’ reasoning, giving students support, and in connecting to students’ experience (Dorrier and Maaß, 2014).
In mathematics, a suitable learning environment can be achieved through tasks that are open in the sense of open approach to mathematics (Samková, 2017), i.e. tasks that have multiple ways of grasping, multiple correct ways of solving, multiple correct answers and/or multiple ways of transforming the task into a new one (Nohda, 2000). The process of solving such tasks consists of various ways of formulating the task mathematically, various ways of interpreting the formulation, of investigating various approaches to the formulated task and the results found, of various ways of interpreting the results, and/or of posing various advanced tasks. Such an environment is rich in alternative reasoning, correct as well as incorrect, based on various expedient ideas (e.g. employing appropriate concepts, generalizing through genuine relations among the concepts, providing logical arguments) or various misconceptional ideas (e.g. employing inappropriate concepts, generalizing through false relations among appropriate concepts, providing illogical arguments).

**Formative assessment**

Formative assessment, as assessment for learning, comprises varied strategies that ought to be implemented into the process of teaching: clarifying and sharing learning intentions and criteria for success, engineering effective classroom discussions and other learning tasks that elicit evidence of student understanding, providing feedback that moves learners forward, activating students as instructional resources for one another, and activating students as the owners of their own learning (Black and Wiliam, 2009). When performed in the classroom, formative assessment may take varied appearances: e.g. of on-the-fly assessment, structured assessment dialogue, peer-assessment, written feedback (Dolin and Evans, 2018).

This paper will address the on-the-fly appearance. In that case, the teacher spontaneously recognizes appropriate opportunities to support students in advancing their learning, and promptly induces a discussion in the classroom (Shavelson et al., 2008). That means that such assessment practice relies on the quality of teacher’s ability to notice, interpret and promote various classroom phenomena that are related to content knowledge of learners. From the perspective of inquiry based mathematics education and open tasks, the teacher ought to be able to recognize and fully understand various ways of grasping the tasks, various solution procedures and various results – not only the commonly used ones but also the uncommon ones (e.g. specific or innovative).

In relation to inquiry based education, scholars usually concur that formative assessment is essential while implementing inquiry based methods of teaching (Harlen, 2013; Dolin and Evans, 2018) and that when properly implemented, formative assessment naturally penetrates the process of inquiry (Hošpesová, 2018). On the other hand, many of future teachers and teachers have never experienced formative assessment as learners, nor have they been trained to implement it in their own teaching (Rokos and Zavodska, 2015).
Concept Cartoons

An educational tool called Concept Cartoons appeared in UK 30 years ago (Keogh and Naylor, 1993), and since then it has been implemented into science and mathematics education at many elementary schools there. Each Concept Cartoon is a simply picture of a bubble dialog among several children, discussing a situation familiar to them from school or everyday reality. Particular bubbles present various alternative opinions on the pictured situation, some of them correct, some incorrect, some might have their correctness unclear or conditioned (see Figures 1, 2, 3). When working with Concept Cartoons in the classroom, the teacher usually asks students to decide which of the pictured children are right and which are wrong, and calls the students for justification of their decisions. Such an arrangement enables the teacher to use Concept Cartoons as a valuable tool in formative assessment (Naylor and Keogh, 2007).

The Concept Cartoons tasks that have two or more correct bubbles can be also considered as open in the sense of open approach to mathematics, since their correct bubbles offer various ways of grasping, various ways of solving and/or various solutions to a mathematics-related problem (Samková and Tichá, 2016). Such a perspective aligns Concept Cartoons with inquiry based mathematics education.

Figure 1: Concept Cartoon on the introductory to the topic of ratios; the template of children with empty bubbles taken from (Dabell, Keogh and Naylor, 2008, no. 2.10)
Figure 2: Concept Cartoon on the part-whole interpretation of fractions; the template of children with empty bubbles taken from (Dabell et al., 2008, no. 3.10)

Figure 3: Concept Cartoon on the introductory to the topic of inequalities; the template of children with empty bubbles taken from (Dabell et al., 2008, no. 1.3)
In this paper, I will render another insight into how Concept Cartoons might be used in relation to formative assessment and inquiry based mathematics education. In my previous research on using Concept Cartoons in university preparation of future primary school teachers, I started to consider each Concept Cartoon as an educational model of a classroom situation, as a representation of school practice that focuses on various content-related comments provided by students and on various ways how teachers may response to them (Samková and Hošpesová, 2016; Samková, 2018). In that sense, Concept Cartoons might help future teachers recognize and fully understand various ways of grasping, various solution procedures and various results that could have been provided by students, and thus create a basis for their own future realization of on-the-fly formative assessment.

As the tool in the study, I created a set of 10 Concept Cartoons related to various topics from primary school mathematics. Five of them were composed as completely new (e.g. Figures 1, 3), the other five were modified from the original set by Dabell et al. (2008) by changing some numbers in the assignment and/or changing the content of some bubbles (e.g. Figure 2). Each of the 10 Concept Cartoons could be considered as open: six of them were with multiple ways of grasping (e.g. Figure 1), nine with multiple correct ways of solving (e.g. Figure 2), three with multiple correct answers (e.g. Figure 3), and four with multiple follow-up tasks (e.g. Figure 3).

The study took place in two lessons in two consecutive weeks. At the first lesson, I assigned the set of Concept Cartoons to the participants, and asked them to respond to the Concept Cartoons in written form: decide which of the pictured children were right and which were wrong, and justify their decisions. They worked on the task individually. The participants were given all the necessary time, so that this stage lasted 70 minutes eventually.

Afterwards, I collected the responses and analysed them qualitatively, using open coding (Miles, Huberman and Saldaña, 2014). During data analysis, I focused on aspect related to on-the-fly formative assessment, i.e. to possible recognition and understanding of various ways of grasping, various solution procedures and various results.

At the second lesson, I returned the responses to the participants, and we started a detailed classroom discussion on the Concept Cartoons as well as on the responses. Again, the participants were given all the necessary time; this stage lasted 90 minutes. During the discussion, I wrote my filed notes on the content of the discussion, and added them to data for another round of qualitative analysis under the same conditions as before.

**Findings**

From the perspective of on-the-fly formative assessment, there were three noticeable aspects related to responses to the Concept Cartoons from Figures 1 to 3:
- The A-bubble in Figure 1 might present an alternative grasping of the pictured situation, based on the idea of both the pictured scales occurring at once and on comparison of the weight of all pictured products (since the text in the bubble is in plural). In that case, the weight of all of the pictured lemons is twice the weight of all of the pictured aubergines and also twice the weight of all of the pictured bananas. This alternative idea was not revealed by any of the participants, though was widely discussed during the second lesson. The discussion led to many questions related to multiple grasping of tasks, i.e. to questions related to various types of ambiguous tasks and to various possible interpretations of their assignments. The participants asked me for other examples of tasks with multiple grasping, started to pose their own examples of situations or word problems that might be grasped variously, provided each other explanations why the posed examples were (not) duly formulated, were curious about possible occurrence of similar tasks in textbooks. Without any impulse on my side, the participants also started the discussion on how they could prepare properly for their own teaching in order to implement various ways of grasping in it, and on how often should they use such tasks in their teaching.

- The bubbles B, C, E in Figure 2 present three different solution procedures leading to determining 3/4 of 12. Most of the participants revealed the content of the C-bubble which consists of the most common procedure used in our country, establishing 3/4 of 12 as 3-times 1/4 of 12. They also revealed the content of the E-bubble which establishes 3/4 via subtracting 1/4 from the whole. But almost all of them failed to reveal the content of the B-bubble which establishes 3/4 of 12 as 1/4 of 3-times 12. Unlike for instance in Germany (Krauthausen and Scherer, 2002), this particular concept is not common in our country. The discussion of this alternative led to multiple questions about various interpretations of fractions, graphical representations, various practically-based word tasks on fractions. Many of the participants commented the B-bubble in the sense that “the result is correct but I do not understand the procedure”, so that the discussion also led to a substantial debate whether the correctness of a result is enough to declare as correct a bubble which shows an unknown procedure leading to this result.

- With the Concept Cartoon from Figure 3, there was no particular bubble that would trigger a debate, but the task as a whole. During the discussion, interesting follow-up questions appeared on how heavy is the flower compared to the cube, whether the number of items on one side of the seesaw is relevant for determining the heaviest/lightest item, or whether the task would be easier with both seesaws (un)balanced.

In summary for all 10 Concept Cartoons, it might be stated that as in the above paragraphs, the issues that triggered the debate were not usually related to a
particular mathematical content (ratio, calculation algorithm, area, data handling, etc.) but rather to the openness of the task. From the perspective of participants, it might be stated that all of the participants took part in the discussion and that they contributed to the discussion more or less regularly and evenly.

Discussion

The findings of this small study enriched the puzzle on “How can we meaningfully employ Concept Cartoons in future teacher education” by another piece of knowledge. They show how Concept Cartoons might be used in preparation of future teachers for better recognizing and understanding of various ways of grasping, various ways of solving and various results, i.e. for supporting future implementation of on-the-fly formative assessment into their school practice. Such an arrangement also effortlessly interconnects formative assessment with inquiry based mathematics education, since suitably created Concept Cartoons can be considered as open tasks and therefore also as inquiry tasks.

In a more general perspective, the text also addresses the issue of various possible formats that representations of school practice may have, namely the formats that can be regarded as a result of a decomposition of practice according to Grossman et al. (2009). Such decompositions involve breaking down practice into its constituent parts for the purposes of teaching and learning. In the particular decomposition consisting of the tool based on Concept Cartoons, the constituent part focuses on various content-related comments provided by students and on various ways how teachers may response to them.

The issues addressed in subsequent discussions on Concept Cartoons relate to the issues that are usually described as the troubled ones from the perspective of teachers (Biton, Hershkovitz, Hoch, Ben-David and Fellus, 2017): dealing with assessment on method as opposed to assessment on results, dealing with missing description of the thinking process and dealing with difficulties in seeing the thinking process that led to the solution. They are also closely related to noticing skills and knowledge based reasoning (Vondrová, 2018).

As for the format of the Concept Cartoon tool itself and the format I propose for its usage in future teacher education, there are two matters that should be addressed here. First, the picture-based environment is not common in research on mathematics education issues and therefore is sometimes considered as unsuitable since the pictures are understood as disturbing and distracting. In this context, there have been interesting recent findings provided by Herbst and Kosko (2013), Friesen and Kuntze (2018) who made a comparison of various formats of representations of school practice (classroom videos, staged videos, animations, comics, vignettes and transcripts) assigned to teachers and future teachers in order to investigate their pedagogical content knowledge. Data analysis showed no significant differences between the formats in relation to the quality of the responses a well as in relation to the difficulty of the task. Second, unlike the
original classroom-discussion format of Concept Cartoons given by Keogh and Naylor (1993), I employ the tool individually and in written format. It appears that with future teachers this new format is valuable, by providing a wide range of information on their pedagogical content knowledge in the written responses (Samková, 2018) or by eliciting subsequent discussions on the written responses (here). Such written format is similar to the one used by Martin et al. (2015) to support formative assessment and learning of mathematics through writing.

The three Concept Cartoons tasks that were discussed in detail in this paper, all belong to practically based problems. Such tasks are a natural source of open and inquiry tasks for mathematics education, since they often have multiple ways of grasping and multiple ways of interpreting the results; sometimes also the classification of the results can be unclear, biased, difficult or even impossible (Koman and Tichá, 1998). Similar properties have also so called ill-structured problems (Fielding-Wells, Dole and Makar, 2014).

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References


### INCLUSIVE MATHEMATICS – IN-SERVICE TRAINING FOR OUT-OF-FIELD TEACHERS

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Abstract

Coping with heterogeneity in mathematics classrooms is a big challenge for teachers. Particularly, considering students with special needs in inclusive classrooms requires further developments of classroom instruction. For teachers’ professional development, in-service courses for mathematics are necessary for both professions, for special education teachers as well as for teachers in regular schools. As many special education teachers are out-of-field teachers for mathematics, a specific program for laying a mathematical and didactical foundation is advisable. In the paper the design of a concrete in-service course is sketched, and corresponding experiences und findings are discussed.

**Keywords**: in-service teacher education, inclusive mathematics, special education

Introduction

In Germany, about 50 % students with special needs on the primary level visit regular schools in inclusive settings (cf. Klemm, 2018). Inclusive classrooms
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