



The  
Fibonacci  
Project

DISSEMINATING INQUIRY-BASED SCIENCE  
AND MATHEMATICS EDUCATION IN EUROPE

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**This learning environment was created within the European project Fibonacci, a project focused on inquiry based science and mathematics education.**

<b>Age:</b>	15 -19 let
<b>Subject:</b>	Mathematics
<b>Topic:</b>	Volume of a cone
<b>Target:</b>	Determine a half of the cone volume.
<b>Form:</b>	Estimates, measurements, calculations, computer models. Working in pairs.
<b>Time needed:</b>	Two lessons, may be separated.
<b>Tools:</b>	Cone glass funnel (diameter 10 cm), 3 plastic transparent cups (200 ml), paper painter's tape (wide 1cm), ruler, bottle of water, Coke, calculator, tables.  Computer with Cabri 3D.
<b>Sources:</b>	L. Samková, Mathematics in a lab, UPVM conference 2011, České Budějovice
<b>Author:</b>	Mgr. Jitka Nováková, L. Samková
<b>Twin Centrum:</b>	TC1 České Budějovice, <a href="http://www.pf.jcu.cz/stru/katedry/m/fibo.html">http://www.pf.jcu.cz/stru/katedry/m/fibo.html</a>

### Cone volume. Worksheet No. 1

Stick a paper tape vertically on a cone funnel. Mark the bottom and the top of the cone on the tape.

- A. Estimate the water level, which corresponds to half of the cone volume. Mark your estimate on the tape. Verify the estimate practically (with help of water and plastic cups).
- B. Estimate the height for  $1/4$ ,  $1/3$ ,  $1/2$ ,  $2/3$  and  $3/4$  of the cone volume, mark estimates on the tape.
- C. Remove the tape carefully and put it on a worksheet. Stick another paper tape on a funnel, mark the bottom and the top on it.
- D. Measure volumes for  $1/4$ ,  $1/3$ ,  $1/2$ ,  $2/3$  and  $3/4$  of the cone in a graduated cylinder, and create a scale on the tape.
- E. Remove the second tape carefully, put it on a worksheet next to the first one, and compare them.

Check all by calculations.

Group No.:

Name:

Put paper tapes here:

Estimates

Measurement

**Cone Volume. Worksheet No. 2**

Determine the half cone level with help of a computer model.  
Change the height and the diameter of the cone.

Group No.:

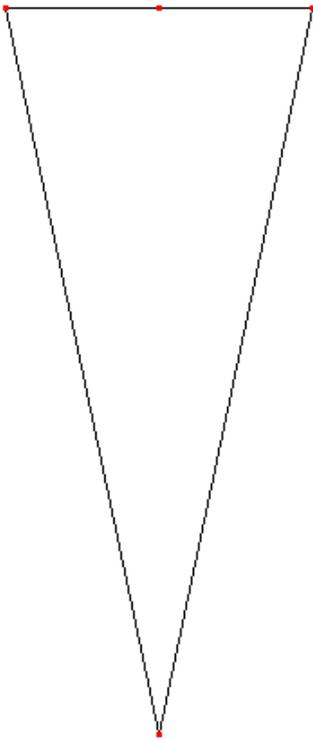
Name:

**Cone volume. Worksheet No. 3**

Here is a side view of a cone. Determine its half-volume level.  
Describe the process.

Group No:

Name:



## Methodological remarks

### Remarks to Worksheet No. 1:

- For every pair of students we prepare a cone funnel, and two paper tapes. The hole at the top of the cone can be plugged by a piece of plasticine.
- Cone funnels are much better than cone flasks, for two reasons:
  - i) Estimating half-volume level when pouring the water off a full cone is much easier with a flask. You can decline the flask at  $90^\circ$ , and the half-volume remains inside. For a funnel, the same declination does not help, it completely empties the cone.
  - ii) The calculation for a cone funnel is much easier and more illustrative – you can better use the similarity of triangles/cones: the whole cone is similar to a cone part filled with water. This does not hold for a cone flask, in this case the part filled with water forms a truncated cone.
- We can motivate students before the A) part of the task with a call for a Coke toast. We fill the funnel with a Coke, and ask students to divide fairly the Coke with a friend: pour a half of the Coke (estimated) to a cup, and the rest of the Coke to another cup. Then compare Coke levels in cups. We may take photographs for them, illustrating the estimate accuracy.
- After estimates and volume measurements, students measure the sizes of the funnel, and verify the results by a calculation. Necessary formulas can be found in tables.

### Remarks to Worksheet No. 2:

- We remind knowledge needed for working with CABRI 3D (functions Length, Volume, Calculator, Text Area).

### Remarks to Worksheet No. 3:

- It is suitable to prepare several variants of the worksheet, with different shapes of cones (different height, different bottom radius).
- Students themselves choose the method: calculations or computer experiment.

### In the end we acquaint the students with required results:

- The level of the water is the third root of its volume ratio. For  $1/2$  of the volume the level equals  $0.79 \cdot \text{height}$  of the cone, that means approximately  $4/5$  of the cone height.
- The level does not depend on particular cone sizes (top angle, bottom radius).
- The same holds for pyramids.

### Our experiences with the project:

- The first phase of the inquiry process – an estimate of a half-volume and its validation – is very important. The result is surprising for students, their estimates are mainly wrong. This discovery is a positive motivation for further investigations.
- The project can be taught as an introduction to cones, as well as a mid-part of the instruction on volumes.