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This learning environment was created within the European project Fibonacci, a project focused on inquiry based science and mathematics education.	
Age:	15-19
Subject:	Mathematics
Topic:	Extreme values of a function
Target:	The use of derivative within a practical problem.
Form:	Practical part – teamwork, theoretical part – individual.
Time needed:	40 min
Tools:	Plasticine, plastic knife, mat, scales, ruler.
Sources:	Borovanský, L. *Ukázky themat daných k pís. maturitním zkouškám z mathematiky na českých středních školách r. 1906* [II.]. (Czech) [Samples of Leaving Examinations in 1906 [mathematics] [II.]]. Časopis pro pěstování matematiky a fysiky, vol. 36 (1907), issue 3, pp. 343-344
Author:	H. Mahnelová
Twin Centre:	TC1 České Budějovice, http://www.pf.jcu.cz/stru/katedry/m/fibo.html

TASK **Practical part**

Here you have a plasticine cylinder of a fixed size. Cut the plasticine off to form a cuboid (= rectangular parallelepiped) of maximum possible volume.

How can you control the increase or decrease of the volume of the emerging solid?

For manual computation of the maximum volume cuboid you need some particular proportions of the original cylinder. Which necessary proportions of the cylinder can be experimentally determined?

Say a hypothesis about the proportions of the cuboid with respect to the proportions of the cylinder.

Hypothesis:

Theoretical part

Determine the proportions of a cuboid inscribed in a cylinder with base radius $2\sqrt{2}$ and height 4, in order to maximize the volume occupied by the cuboid.

METHODOLOGICAL COMMENT

In the first part, students work in groups, discuss, and create a hypothesis.

The decrease or increase of the volume of the plasticine solid they can control by a weight change of the solid, since the volume is directly proportional to the weight.

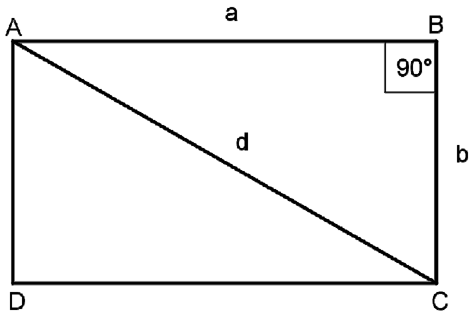
In the second part, students individually verify the correctness of their hypothesis in one particular case.

The work is focused on an application of derivatives of one-variable functions in the process of finding the global extreme of the function. It is assumed that the student knows the concept of derivative.

Smarter students manage to realize that the volume of a cuboid is directly proportional to the area of its base, and therefore they look for a global maximum of the area of the base as a function of the length of one of base sides.

An example of correct solution to a theoretical part:

Suppose that base proportions of the cuboid are *a*, *b*, the height of the cuboid *h*.
Then the cuboid volume equals $V = a \cdot b \cdot h$.
In our particular case, $h = 4$, and $V = 4ab$.
Moreover, the length of the diagonal of the cuboid base equals the diameter of the cylinder base, that means $d = |AC| = 2r$, in our case $d = 2 \cdot 2\sqrt{2} = 4\sqrt{2}$.



The Pythagorean theorem gives $a^2 + b^2 = d^2$, that means $a^2 + b^2 = 32$, thus $b = \sqrt{32 - a^2}$.
Together with the formula above it gives $V = 4a\sqrt{32 - a^2}$.
Let's find extrems of the volume function $V(a) = 4a\sqrt{32 - a^2}$.

The first derivative equals $V^{/}(a) = 4 \cdot \dfrac{32 - 2a^2}{\sqrt{32 - a^2}}$, the second derivative equals $V^{//}(a) = 4 \cdot \frac{2a^3 - 128a + 32}{(32 - a^2)\sqrt{32 - a^2}}$.

Further, $V^{/}(a) = 0 \Leftrightarrow 32 - 2a^2 = 0$, that means $a = 4$, ($a > 0$).

Since $V^{//}(4) = -\frac{352}{16} < 0$, the function $V(a)$ reaches its maximum at the point $a = 4$.

Finally, we have to calculate b, $b = \sqrt{32 - a^2} = \sqrt{16} = 4$.
The desired cuboid is a cube, since $a = b = h = 4$.